

Reading Group (Fall 2022, uOttawa) — Toric Varieties and its Applications

Rui Xiong (rxion043@uOttawa.ca)

This is a rough plan for the reading group “[Toric Varieties and its Applications](#)”. The final purpose is to understand the proof of [Read’s conjecture](#) stating that the absolute value of coefficients of the [chromatic polynomial of a graph is unimodal \(sinkless\)](#). The proof is established in a series of papers with main ideas originally from [algebraic geometry](#), e.g.

- June Huh, Milnor numbers of projective hypersurfaces and the chromatic polynomial of graphs, J. Amer. Math. Soc. [arXiv:1008.4749]
- June Huh, Eric Katz, Log-concavity of characteristic polynomials and the Bergman fan of matroids 2011, Mathematische Annalen. [arXiv:1104.2519]
- Karim A. Adiprasito, June Huh, Eric Katz. Hodge theory for combinatorial geometries, Annals Of Mathematics 2015. [arXiv:1511.02888]

See also the following survey

- Eric Katz, Matroid theory for algebraic geometers. [arXiv:1409.3503]

Note that June Huh was awarded the [Fields Medal](#) in 2022 due to the mentioned work and more work in this direction.

The main geometric object is toric variety, a sort of algebraic variety parametrized by combinatorial objects — fans. It provides many examples (and counterexamples) in algebraic geometry. The standard book of toric variety is

- W. Fulton. Introduction to Toric Varieties.

Most of the combinatorial applications (including the proof we promised) of toric variety use [Hodge theory](#). For general information on combinatorial applications, we recommend

- R.P. Stanley. Combinatorial Applications of the Hard Lefschetz Theorem.
- J. Huh. Combinatorial Applications of the Hodge–Riemann Relations.

My plan is the following,

- I will give the talk for each meet except when any audience wants to share. Actually, it seems to me that a detailed note directed to this topic is not yet written.

In other words, you are not supposed to give any talk.

- To maximize the possible achievement each time, I will first review the algebraic geometry I will use later.

In other words, if you are combinatorics-allergic and literally lost in the discussion, at least, you would know some algebraic geometry.

- There will be an [offline](#) talk each week. The exact time and room will be announced later. It would probably be the [unit just before Joint Ottawa/Carleton Algebra Seminar](#).
- We need to require basic knowledge of algebra geometry, for example,

Try to figure out which ring R such that $\text{Spec } R = \mathbb{C} \setminus \{0\}$.

Actually, we will mainly use the complex algebraic geometry, so not absolutely the style of Hartshorne.

If you are willing to join or have questions, please contact me: rxion043@uOttawa.ca.